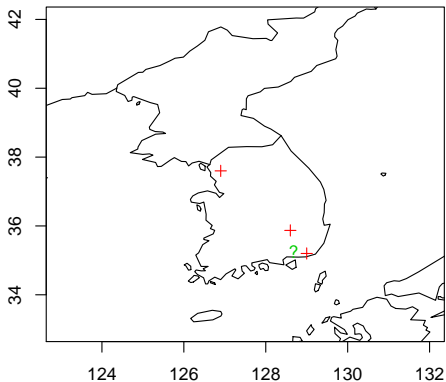




## SPATIAL OR SPATIO-TEMPORAL STATISTICS

- Spatial or spatio-temporal statistics concern with accounting for spatial (or spatio-temporal) dependence



## SPATIAL OR SPATIO-TEMPORAL STATISTICS

- Suppose  $X_1, X_2, X_3$  are data observed from three locations and we want to predict  $X_0$
- The “best” estimate (predicted value) for  $X_0$  would be the average of  $X_1, \dots, X_3$ ,

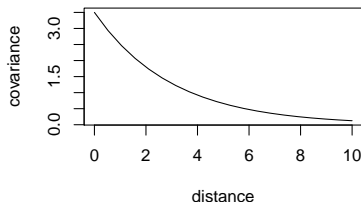
$$\frac{1}{3}(X_1 + \dots + X_3),$$

if  $X_1, X_2, X_3$  are independent

- If some of  $X_1, \dots, X_3$  (and  $X_0$ ) are “correlated”, we should give different weights for the estimate:  $a_1X_1 + a_2X_2 + a_3X_3$

## SPATIAL OR SPATIO-TEMPORAL STATISTICS

- In spatial statistics, the main focus is how to model such dependence
- Basic model assumes “distance-based dependence”



- Nonstationarity:
  - ▶ Land/sea
  - ▶ Latitude dependence
  - ▶ Longitude dependence
  - ▶ Dependence on elevation
  - ▶ ...
- Spatial dependence vs cross-dependence for multivariate spatial data

# STATISTICAL ANALYSIS WITH CLIMATE MODEL OUTPUTS

- Each climate model output at a given time (or temporal average) is considered as spatial data
- Multi-model ensembles are considered as spatial data that are dependent on each other
  - ▶ At a given time, given space, values from two different models can be correlated
- Ensemble outputs from a climate model are usually considered independent
- Even for a given climate model, one might want to consider dependence of different climate variables (e.g. surface temperature and precipitation)

## DATA CONSIDERED

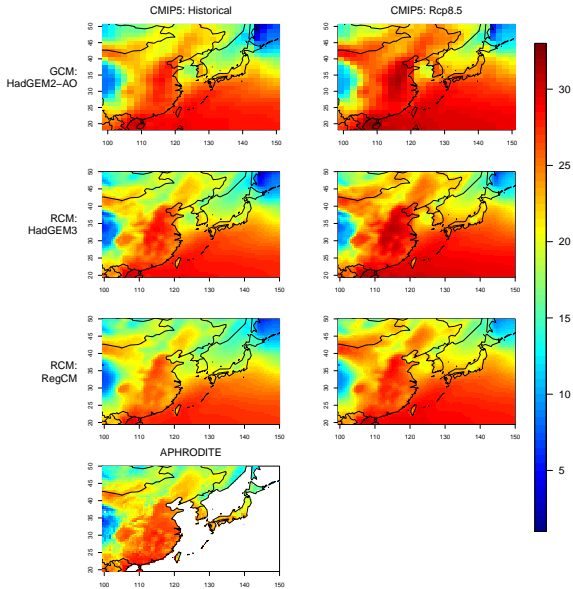
- CORDEX outputs over some part of East Asian domain (around Korean peninsula)
- Two regional climate models considered (HadGEM3-RA and RegCM4, driven by the same GCM, HadGEM2-AO)
- Surface temperature (DJF and JJA averages)
- Historical runs (1979 to 2005) and future projections with RCP8.5 (2024 to 2050)
- Interpolated to a common grid of  $0.5^\circ \times 0.5^\circ$
- “Observations” used: APHRODITE for 1979-2005

## OBJECTIVE

- Assess the effect of “climate models”, “time”, “interaction” on the surface temperature (as well as estimate the trend)
- **Functional ANOVA model:**

$$Y_{ijt}(\mathbf{s}) = \mu(\mathbf{s}) + i \alpha(\mathbf{s}) + j \beta(\mathbf{s}) + ij (\alpha\beta)(\mathbf{s}) + \gamma(t - 14) + \epsilon_{ijt}(\mathbf{s})$$

- ▶  $i = -1$  for **HadGEM3-RA** and  $i = 1$  for **RegCM4**
  - ▶  $j = -1$  for **current** and  $j = 1$  for **future** run
  - ▶  $t = 1, \dots, 27$
  - ▶ E.g.  $2\alpha(\mathbf{s})$  gives difference of RegCM4 and HadGEM3-RA at a given location  $\mathbf{s}$
- Can be naturally extended to other climate variables, multiple regional climate models, and other domains



**FIGURE :** Spatial map of surface air temperatures (JJA, unit: °C) by GCM (HadGEM2-AO) and two RCMs (HadGEM3-RA and RegCM4) for the historical (left) and RCP8.5 (right) scenario experiments.

## SPATIAL MODEL FOR $\mu$

- We write

$$\mu(\mathbf{s}) = \begin{cases} \mu_0 + \sigma_\mu e^\eta W(\mathbf{s}), & \text{if } \mathbf{s} \in \text{land,} \\ \mu_0 + \sigma_\mu W(\mathbf{s}), & \text{if } \mathbf{s} \in \text{sea,} \end{cases}$$

- $W$  is a mean zero *Gaussian process*
- Covariance structure (i.e. spatial dependence) of  $W$  given by a *Matérn correlation function* that only depends on distance
- For the mean of  $\mu$ :

$$\mu_0(\mathbf{s}) = \begin{cases} \mu_{00} + \mu_{01} \sin\{L(\mathbf{s})\} + \mu_{02} + \mu_{03}A(\mathbf{s}), & \text{if } \mathbf{s} \in \text{land} \\ \mu_{00} + \mu_{01} \sin\{L(\mathbf{s})\}, & \text{if } \mathbf{s} \in \text{sea.} \end{cases}$$

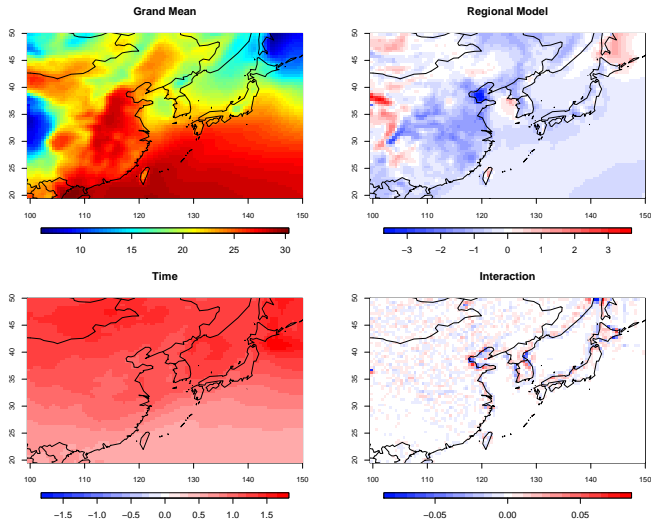
( $L$ : latitude,  $A$ : elevation)

- Parameters to be estimated:  $\mu_{0x}$ ,  $\gamma$ ,  $\sigma_\mu$ ,  $\eta$ , mean parameters of  $\beta$  (latitude dependence), covariance parameters of  $W$ ,  $\alpha$ ,  $\beta$ , and ( $\alpha\beta$ )

# STATISTICAL MODEL FITTING

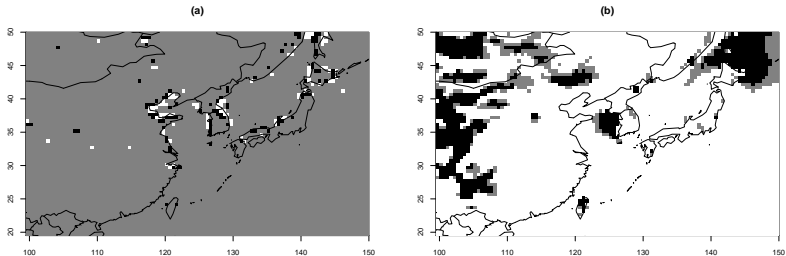
- One could use either *Likelihood method* or *Bayesian method* to fit the models
- In our paper, we used Bayesian MCMC procedure to get *posterior samples* of each parameters, which gives distributions (uncertainties) of each parameter
- Separate models (with same structure) are fitted for JJA and DJF
- Details of *priors* and model fitting procedure are given in the paper

# RESULTS FOR JJA



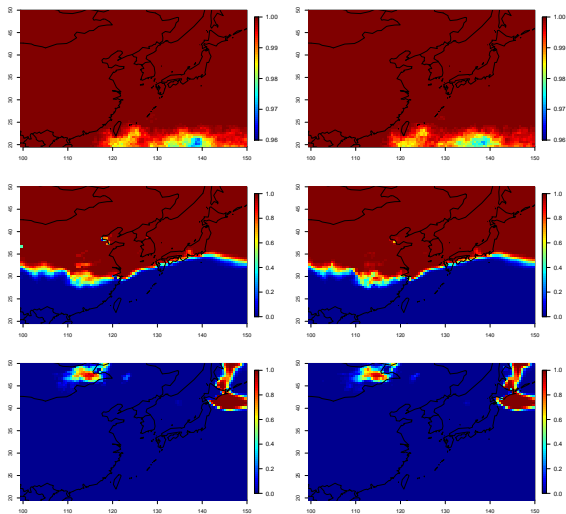
**FIGURE :** Posterior means of  $\mu$  (“Grand Mean”),  $\alpha$  (“Regional Model”),  $\beta$  (“Time”), and  $(\alpha\beta)$  (“Interaction”) (unit:  $^{\circ}\text{C}$ ).

# RESULTS FOR JJA



**FIGURE :** (a) Regions where  $(\alpha\beta)$  is credibly different from zero (black regions for positive  $(\alpha\beta)$  and white regions for negative  $(\alpha\beta)$ ). (b) Regions where  $\alpha$  is credibly different from zero

# RESULTS FOR JJA



**FIGURE :** Posterior probabilities that temperature (JJA case) increases more than 1°C (top), 2°C (middle), and 3°C (bottom) from RegCM4 model (left) and HadGEM3-RA model (right).

## RESULTS FOR JJA

- $2\beta$  (which represents the increase of temperature from current to future) is around  $1 - 3^\circ\text{C}$  ( $2\beta + 2(\alpha\beta)$  gives the same increase by RegCM4)
- Latitude plays an important role in temperature increase from current to future
- Area averaged value of  $\beta$  for JJA is 1.06 and that for DJF is 0.97

## RESULTS REGARDING TREND ( $\gamma$ )

- $2\beta$  gives temperature increase from current to future
- $\gamma$  gives linear trend in temperature (per year)
- $\gamma$  is assumed fixed over space

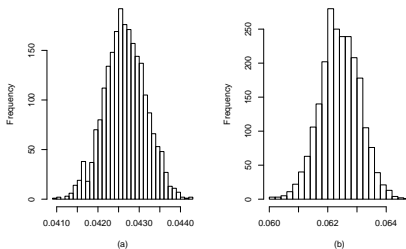


FIGURE : Posterior samples of  $\gamma$ . (a) JJA, (b) DJF.

## RESULTS REGARDING TREND ( $\gamma$ )

- Temperature increase compared to the summer of 1979:

$$\gamma(x - 2024) + 2\beta, \quad x \in (2024, 2050)$$

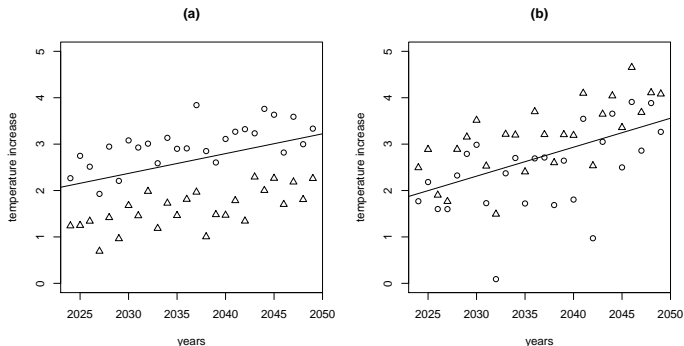


FIGURE : Circles from HadGEM3-RA and triangles from RegCM4.

# DISCUSSIONS

- Computational technique is used to deal with large size of data
- Can be applied to any domain, any climate variables
- Can be extended to multiple climate variables at the same time
- More details in the paper: Asia-Pac. J. Atmos. Sci. 52(2)  
237-249