

**Modeling climate variability as anomalies of  
quasi-stationary PDF moments**

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Unavoidable uncertainties in  
climate predications:

→ Stochastic and dynamic  
modeling

→ **Fokker Planck equation for complete PDF  
prediction, which is an approach too difficult, if  
ever possible!**

# **Ensemble Forecasting for Climate Variability**

**Stochastic-dynamic forecasting approach (not practical) Epstein (1969).**

**Monte-Carlo forecasting method (Leith,1974).**

**...**

**Existing ensemble forecasting method is adopted from NWP.**

**Question:**

**Alternatives?**

# **Climate variability**

**Climate is normally defined by statistic measures such as long-term mean and standard deviations or is defined by the first a few (two) moments of associated with PDFs of physical variables.**

**Climate variability, such as anomalies in seasonal-mean fields and in standard deviations of daily weather variability, may be viewed as the anomalies of the first and second moments of the PDFs. This view is based on the assumption that the climate system can be considered as a quasi-stationary stochastic system.**

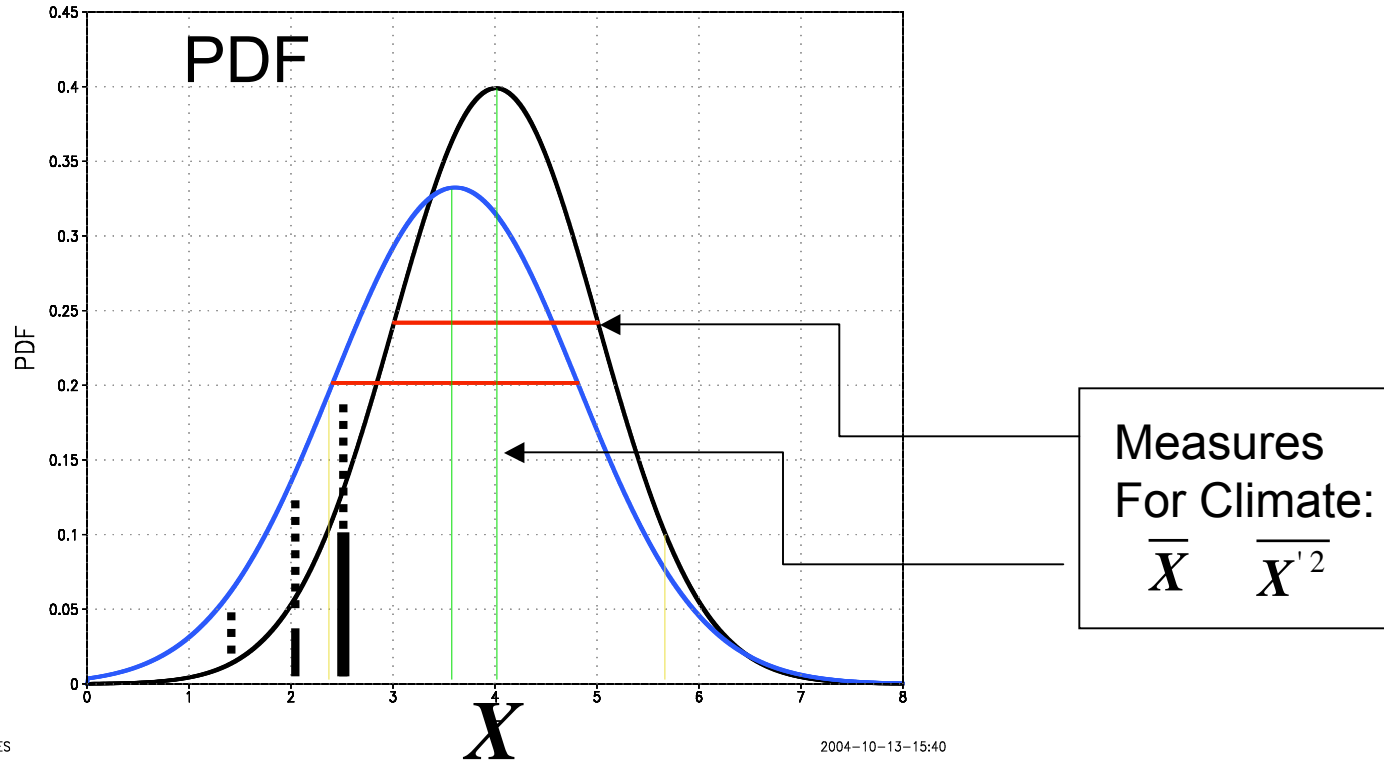
## Quasi-ergodic assumption

$$\overline{X} \approx \langle X \rangle$$

Namely: “long” time mean of a stochastic realization can be approximated by ensemble mean

Then, predicting climate variability may become the question:

Can we directly predict the first and second (or even third) moments of the PDF of the quasi-stationary stochastic climate system?



Climate variability:  $\Delta \bar{X} = \bar{X}_a, \Delta \overline{X^2}, \dots$

Can we directly formulate dynamic model for the first and second (or even third) moments of the PDF?

## **An new approach:**

The climatological parts of the first and second moments of PDFs are prescribed by the observed history of the climate system, in other words, we prescribe the stationary part of the stochastic climate system.

A dynamic system for anomalies in the first and second is then formulated utilizing the GCM framework under closure approximations.

## **Expectations:**

This dynamic anomaly framework for ensemble-mean modeling may become a useful new tool for predictions of climate variability.

## Background Stochastic Stationary Ensemble

$$X_c = \bar{X}_c(\lambda, \varphi, z) + X'_c(\lambda, \varphi, z, t)$$

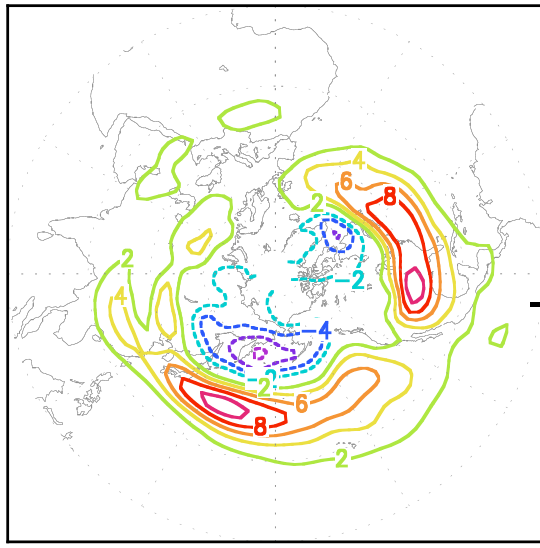
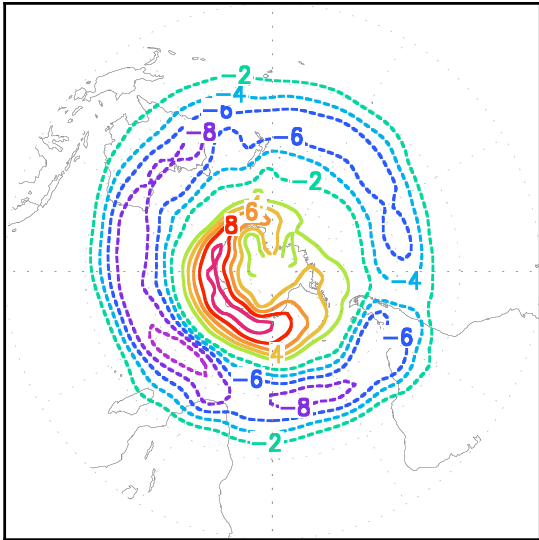
$$X'_c = \sum_{n=1}^{N_c} \sigma_n \xi_n(t) E_n(\lambda, \varphi, z, t) + cc$$

Anomaly Stochastic Ensemble:  $X_a = X - X_c$

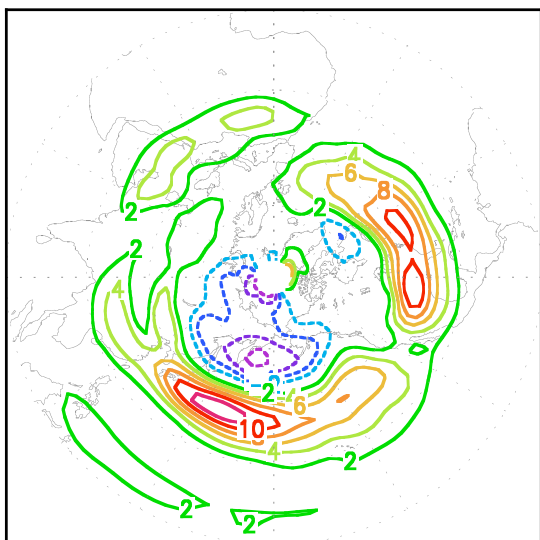
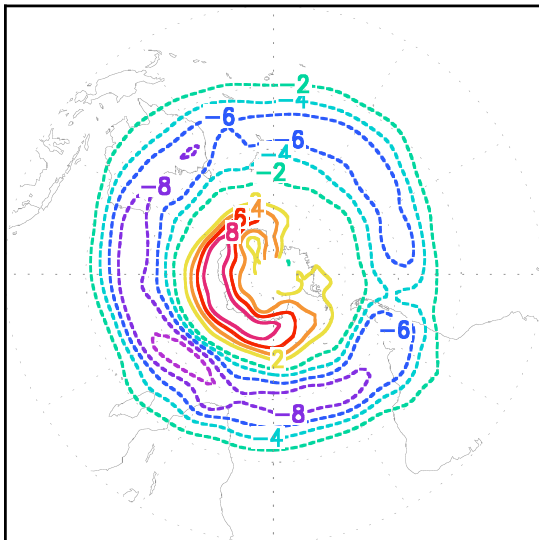
Linear EMD under the second order closure  
(Jin et al 2006, Jin and Lin 2007)

$$\begin{aligned} \frac{\partial \langle X_a \rangle}{\partial t} + L \langle X_a \rangle &= \langle Q_a \rangle - \langle ND(X'_c, X'_a) \rangle \\ &= \langle Q_a \rangle - L_f \langle X_a \rangle \end{aligned}$$

$L_f$ : Synoptic eddy and low-frequency flow (SELF)  
feedback through nonlinear heat and moment advections

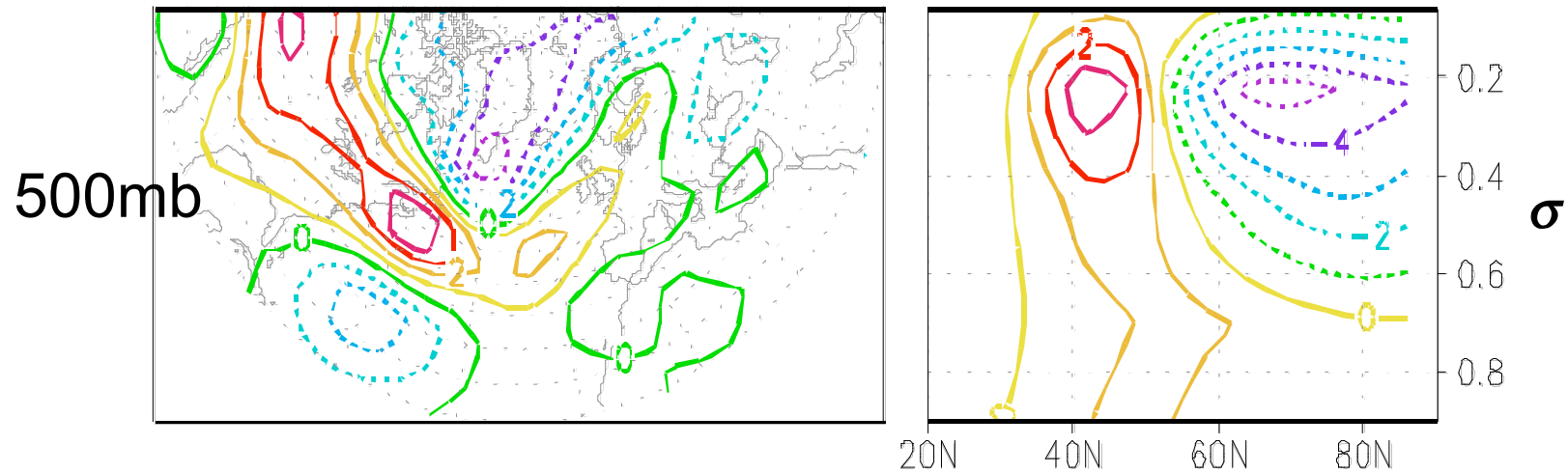
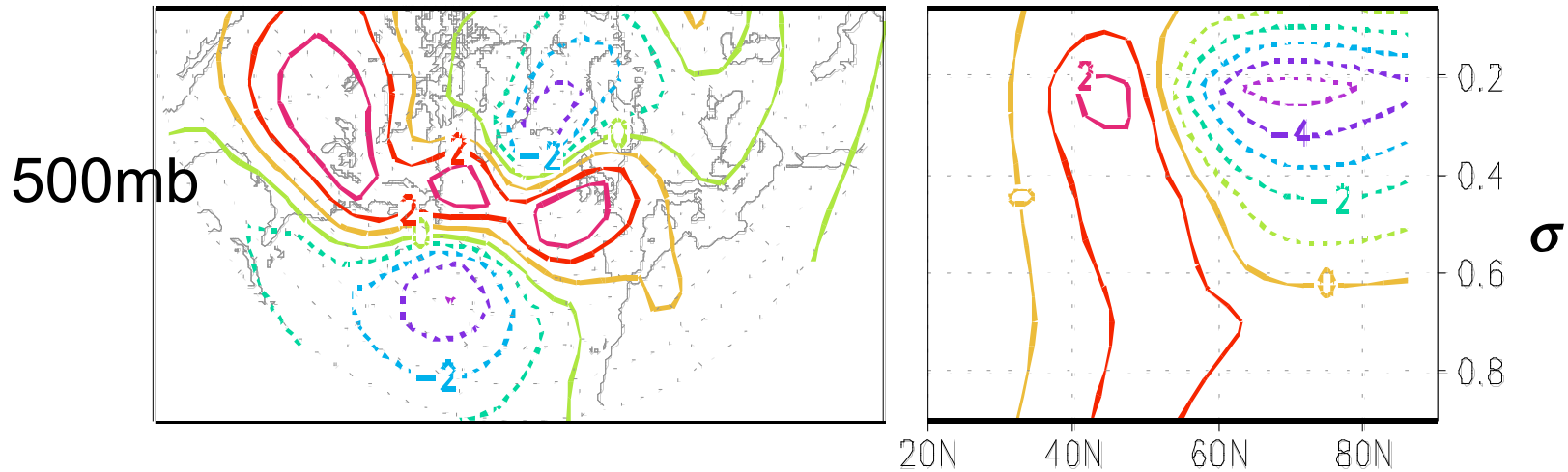


$-\Delta^{-1}(\nabla \cdot (\bar{\mathbf{v}}' \zeta'))$   
Observed



Reconstructed

## Observed eddy forcing related to NAO



## Simulated eddy forcing related to NAO

by the new system based on a linear 5-layer-T21 PE model

Key undertaking is to formulate the SELF feedbacks in the physical processes of the climate models, for instance:

$$Q_a \approx G(X_c + X'_c)X_a + B(X_c + X'_c)F_a$$

$$\langle Q_a \rangle \approx \langle G(X_c + X'_c) \rangle \langle X_a \rangle + \frac{\partial G(X_c)}{\partial X} \langle X'_c X_a \rangle + \dots$$

$$+ \langle B(X_c + X'_c) \rangle F_a$$



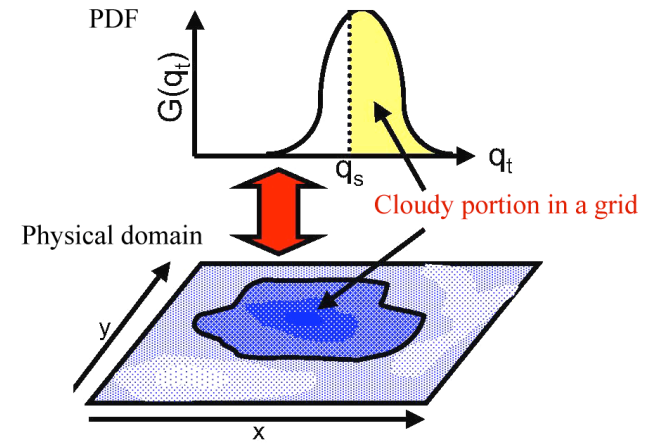
SELF feedback in physical processes

# Large-scale condensation

'fast-condensation' adjustment

$$Q'_h{}^L = \frac{L}{\tau_l c_p} q'_l \quad , \quad Q'_q{}^L = -\frac{1}{\tau_l} q'_l$$

$q'_l$  is a perturbation cloud water as determined by linearizing the subgrid-scale PDF of LeTreat and Li (1991).



$$q'_l = \begin{cases} q' - \frac{c_p}{L} \gamma T' & \text{for } (1-b)\bar{q} > q_s(\bar{T}) \\ \mathcal{A}' & \text{for } (1-b)\bar{q} \leq q_s(\bar{T}) < (1+b)\bar{q} \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{A}' = \frac{d\mathcal{A}}{dq} q' = \frac{\Delta\bar{q}}{2b\bar{q}} \left[ (1+b)q' - \frac{c_p}{L} \gamma T' \right] - \frac{\Delta\bar{q}^2}{4b\bar{q}^2} q'$$

$$\Delta\bar{q} = (1+b)\bar{q} - q_s(\bar{T})$$

$$\mathcal{A} = \frac{1}{2bq_w} \int_{q_s}^{(1+b)q_w} (\hat{q}_w - q_s) d\hat{q}_w = \frac{1}{4bq_w} [(1+b)q_w - q_s]^2$$

$b$  is only the parameter which determines the PDF width (ordinary  $b=0.1-0.2$  ).

## **Summary:**

**A new framework is proposed for modeling (predicting) climate variability as anomalies in the first(and second) moments of PDF.**

**Its usefulness is yet to be demonstrated by conducting hindcast and forecast experiments.**